Limits, Continuity and Differentiability

Question1

If $f(x)=\frac{x(a^x-1)}{1-\cos x}$ **and** $g(x)=\frac{x(1-a^x)}{a^x\left(\sqrt{1-x^2}-\sqrt{1+x^2}\right)}$**, then** $\lim_{x\to 0}(f(x)-g(x))=$

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Options:

A.

$3\log a$

B.

e^a

C.

$2\log a$

D.

$\log a$

Answer: D

Solution:

Given, $f(x)=\dfrac{x\left(a^x-1\right)}{1-\cos x}$

And $g(x)=\dfrac{x\left(1-a^x\right)}{a^x\left(\sqrt{1-x^2}-\sqrt{1+x^2}\right)}$

$$\therefore \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{x\left(a^x - 1\right)}{1 - \cos x}$$

$$= \lim_{x\to 0} \frac{x \frac{\left(a^x - 1\right)}{x}}{\frac{1 - \cos x}{x}}$$

$$= \lim_{x\to 0} \frac{a^x - 1}{x} \times \frac{x^2}{1 - \cos x}$$

$$= \lim_{x\to 0} \frac{a^x - 1}{x} \times \lim_{x\to 0} \frac{x^2}{1 + \cos x}$$

$= \log a \times 2$

$= 2 \log a$

Also, $\lim_{x\to 0} g(x)$

$$= \lim_{x\to 0} \frac{x\left(1 - a^x\right)}{a^x\left(\sqrt{1 - x^2} - \sqrt{1 + x^2}\right)}$$

$$= \lim_{x\to 0} \frac{x\left(1 - a^x\right)\left(\sqrt{1 - x^2} + \sqrt{1 + x^2}\right)}{a^x\left(\sqrt{1 - x^2} - \sqrt{1 + x^2}\right)}$$

$$\left(\sqrt{1 - x^2} + \sqrt{1 + x^2}\right)$$

$$= \lim_{x\to 0} \frac{x\left(1 - a^x\right)\left(\sqrt{1 - x^2} + \sqrt{1 - x^2}\right)}{a^x\left(1 - x^2 - 1 + x^2\right)}$$

$$= \lim_{x\to 0} \frac{x\left(1 - a^x\right)\left(\sqrt{1 - x^2} + \sqrt{1 - x^2}\right)}{a^x\left(-2x^2\right)}$$

$$= \lim_{x\to 0} \frac{\left(a^x - 1\right)\left(\sqrt{1 - x^2} + \sqrt{1 - x^2}\right)}{a^x(2x)}$$

$$= \lim_{x\to 0} \frac{a^x - 1}{x} \times \lim_{x\to 0} \frac{\sqrt{1 - x^2} + \sqrt{1 - x^2}}{2a^x}$$

$$= \log a \times \frac{1 + 1}{2(1)}$$

$= \log_a a(1) = \log a$

$\therefore \lim_{x\to 0}(f(x) - g(x))$

$= \lim_{x\to 0} f(x) - \lim_{x\to 0} g(x)$

$= 2 \log a - \log a = \log a$

Question2

If $f(x) = \begin{cases} \frac{a \sin x - bx + cx^2 + x^3}{2 \log(1+x) - 2x^3 + x^4} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

is continuous at $x = 0$**, then**

Options:

A.

$a = 2b$

B.

$a = b$

C.

$a = b = c$

D.

$b = c$

Answer: B

Solution:

Given,

$$f(x) = \begin{cases} \frac{a \sin x - bx + cx^2 + x^3}{2 \log(1+x) - 2x^3 + x^4}, x \neq 0 \\ 0, \quad x = 0 \end{cases}$$

is continuous at $x = 0$

$$\therefore \lim_{x \to 0} f(x) = f(0).$$

$$\Rightarrow \lim_{x \to 0} \frac{a \sin x - bx + cx^2 + x^3}{2 \log(1+x) - 2x^3 + x^4} \left(\frac{0}{0}\right) = f(0)$$

$$\Rightarrow \lim_{x \to 0} \frac{a \cos x - b + 2cx + 3x^2}{\frac{2}{(1+x)} - 6x^2 + 4x^3} = 0$$

$$\Rightarrow \quad \frac{a-b}{2} = 0 \Rightarrow a - b = 0$$

$$\Rightarrow \quad a = b$$

Question3

If the function $g(x)=\begin{cases}K\sqrt{x+1} & ,0\leq x\leq 3\\ mx+2 & ,3<x\leq 5\end{cases}$ **is differentiable, then** $K+m=$

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Options:

A.

4

B.

2

C.

6

D.

0

Answer: B

Solution:

Given, $g(x)=\begin{cases}K\sqrt{x+1}, & 0\leq x\leq 3\\ mx+2, & 3<x\leq 5\end{cases}$ is differentiable

Since, $g(x)$ is differentiable therefore it is continuous.

at $x=3$

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)$$
$$\Rightarrow \lim_{h\to 0} f(3-h)=\lim_{h\to 0} f(3+h)$$
$$\Rightarrow \lim_{h\to 0}(K\sqrt{(3-h+1)})=\lim_{h\to 0}(m(3+h)+2)$$
$$\Rightarrow K\sqrt{4}=m(3)+2$$

$\Rightarrow \quad 2K = 3m + 2 \qquad \ldots (i)$

for $0 \leq x \leq 3$

$g'(x) = K\frac{1}{2\sqrt{x+1}}$

And for $3

$g'(x) = m$

for differentiability at $x = 3$

$$\begin{aligned} & Lg'(3) = Rg'(3) \\ & \Rightarrow \frac{K}{2\sqrt{4}} = m \\ \Rightarrow & K = 4m \qquad \ldots (ii) \end{aligned}$$

On Solving Eqs. (i) and (ii), we get

$$\begin{aligned} & 2(4m) = 3m + 2 \\ \Rightarrow \quad & 8m = 3m + 2 \\ \Rightarrow \quad & 5m = 2 \Rightarrow m = \frac{2}{5} \end{aligned}$$

And $\quad K = \frac{8}{5}$

$\therefore \quad K + m = \frac{8}{5} + \frac{2}{5} = \frac{10}{5} = 2$

Question4

If $[x]$ is the greatest integer function, then

$\lim\limits_{x \to 3} \frac{(3-|x|+\sin|3-x|)\cos[9-3x]}{|3-x|[3x-9]}$

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Options:

A.

0

B.

1

C.

2

D.

-2

Answer: D

Solution:

Since, $x \rightarrow 3^{-}$, we have $x<3$.

$\therefore |x|=x$ and $|3-x|=3-x$

As, $x \rightarrow 3^{-}, 3x \rightarrow 9^{-}$, so $9-3x \rightarrow 0^{+}$

Thus, $[9-3x]=0$ for x slightly less than 3 and $[3x-9]=-1$ for x slightly less than 3 .

$$\therefore \lim_{x \rightarrow 3^{-}} \frac{(3-x+\sin(3-x)) \cos(0)}{(3-x)(-1)} = \lim_{x \rightarrow 3^{-}} \frac{(3-x+\sin(3-x))}{(3-x)(-1)}$$

Let $t=3-x$.

As, $x \rightarrow 3^{-}, t \rightarrow 0^{+}$

$$\text{So,} \quad \lim_{x \rightarrow 3^{-}} \frac{(3-x+\sin(3-x))}{(3-x)(-1)}$$

$$\Rightarrow \lim_{t \rightarrow 0^{+}} \frac{t+\sin t}{-t}$$

$$\Rightarrow \lim_{t \rightarrow 0^{+}}\left(-1-\frac{\sin t}{t}\right)$$

$$\Rightarrow -1-1 \quad \left[\because \lim_{t \rightarrow 0} \frac{\sin t}{t}=1\right]$$

$$\Rightarrow -2$$

$$\therefore \lim_{x \rightarrow 3^{-}} \frac{(3-|x|+\sin|3-x|) \cos[9-3x]}{|3-x|[3x-9]} = -2$$

Question5

Let ' a ' be a positive real number. If a real valued function

$$f(x)=\begin{cases}\frac{6^x-3^x-2^x+1}{1-\cos\left(\frac{x}{a}\right)} & \text{if } x\neq 0\\ \log 3\log 4 & \text{if } x=0\end{cases}$$ **is continuous at $x=0$, then $a=$**

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Options:

A.

1

B.

2

C.

3

D.

4

Answer: A

Solution:

Given,

$$f(x)=\begin{cases}\frac{6^x-3^x-2^x+1}{1-\cos\left(\frac{x}{a}\right)}, & x\neq 0\\ \log 3\log 4, & x=0\end{cases}$$

Now, $f(0)=\log 3\log 4$

Now, $\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{1-\cos\left(\frac{x}{a}\right)}$

$$\Rightarrow \lim_{x\to 0}\frac{(3^x-1)(2^x-1)}{1-\cos\left(\frac{x}{a}\right)}$$

$$\Rightarrow \lim_{x\to 0}\left[\frac{\frac{3^x-1}{x}\cdot\frac{2^x-1}{x}}{\frac{1-\cos\left(\frac{x}{a}\right)}{x^2}}\right]$$

$$\Rightarrow \frac{(\ln 3)(\ln 2)}{\frac{1}{(2a^2)}}\left[\begin{array}{l}\because \lim_{x\to 0}\frac{a^x-1}{x}=\ln a\\ \text{and } \lim_{x\to 0}\frac{1-\cos kx}{x^2}=\frac{k^2}{2}\end{array}\right]$$

$$=2a^2(\ln 3)\ln(2)$$

Since, given function is continuous

So, $\lim_{x \to 0} f(x) = f(0)$

$$\begin{aligned} \Rightarrow \quad & 2a^2(\ln 3)(\ln 2) = (\ln 3)(\ln 4) \\ \Rightarrow \quad & (\ln 3)(\ln 2)^2 = 2(\ln 3)(\ln 2) \\ \Rightarrow \quad & 2a^2 = 2 \Rightarrow a^2 = 1 \end{aligned}$$

$\Rightarrow \quad a = 1 \quad (\because a$ is a positive real number)

Question6

$$\lim_{x \to \frac{3}{2}} \frac{(4x^2 - 6x)(4x^2 + 6x + 9)}{\sqrt[3]{2x} - \sqrt[3]{3}} =$$

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Options:

A. $\sqrt[3]{3^{17}}$

B. $\sqrt[3]{3^{16}}$

C. $\sqrt[3]{3^{15}}$

D. $\sqrt[3]{3^{14}}$

Answer: A

Solution:

$$\lim_{x \to \frac{3}{2}} \frac{2x(2x-3)(4x^2+6x+9)}{(2x)^{1/3} - 3^{1/3}}$$

$$\lim_{x \to \frac{3}{2}} 2x \left| \frac{8x^3 - 27}{(2x)^{1/3} - 3^{1/3}} \right|$$

$$3 \lim_{x \to \frac{3}{2}} \frac{24x^2}{\frac{1}{3}(2x)^{-\frac{2}{3}} \cdot 2} = \frac{72 \times 3\left(\frac{3}{2}\right)^2}{3^{-\frac{2}{3}}}$$

$$= 72 \times 3 \times \frac{9}{4} \times 3^{\frac{2}{3}} \times \frac{1}{2}$$

$$= \frac{1}{2} \times 18 \times 27 \times 3^{2/3} = 9 \times 27 \times 3^{2/3}$$

$$= 3^{2+3+\frac{2}{3}} = 3^{\frac{17}{3}} = \sqrt[3]{3^{17}}$$

Question7

If the real valued function $f(x) = \int \frac{(4^x - 1)^4 \cot(x \log 4)}{\sin(x \log 4) \log(1 + x^2 \log 4)}$, if $x \neq 0$ is continuous at $x = 0$, then $e^k =$

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Options:

A. 1

B. 4

C. e

D. 2

Answer: B

Solution:

To evaluate the limit of the function $f(x)$ as x approaches 0, we start with the given expression and simplify step-by-step:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(4^x - 1)^4 \cot(x \ln 4)}{\sin(x \ln 4) \ln(1 + x^2 \ln 4)}$$

First, rewrite the terms using a convenient limit form:

$$= \lim_{x \to 0} \frac{\left(\frac{4^x - 1}{x}\right)^4 x^4 \cos(x \ln 4)}{\left(\frac{\sin(x \ln 4)}{x \ln 4}\right)^2 x^2 (\ln 4)^2} \times \frac{\ln(1 + x^2 \ln 4)}{x^2 \ln 4} \times x^2 \ln 4$$

Here, consider the limit:

$$\lim_{x \to 0} \frac{4^x - 1}{x} = \ln 4$$

$$\lim_{x \to 0} \frac{\sin(x \ln 4)}{x \ln 4} = 1$$

$$\lim_{x \to 0} \frac{\ln(1 + x^2 \ln 4)}{x^2 \ln 4} = 1$$

Substituting these values into the expression:

$$= \lim_{x \to 0} \frac{(\ln 4)^4}{(\ln 4)^3} = \ln 4 = k$$

Thus, the value of e^k is:

$$e^k = e^{\ln 4} = 4$$

Question8

If $0 \leq x \leq \frac{\pi}{2}$, then $\lim_{x \to a} \frac{|2\cos x - 1|}{2\cos x - 1}$

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Options:

A. does not exist at all points in $\left[0, \frac{\pi}{2}\right]$

B. $= 1$, when $a = \frac{\pi}{3}$

C. $= -1$, when $a = \frac{\pi}{3}$

D. $= 1$, when $0 \leq a < \frac{\pi}{3}$

Answer: D

Solution:

If $x \in \left[0, \frac{\pi}{3}\right) \cos x > \frac{1}{2}$

then, $\lim_{x \to a} = \frac{(2\cos x - 1)}{2\cos x - 1} = 1$

Question9

The real valued function $f(x) = \frac{|x-a|}{x-a}$ is

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Options:

A. continuous only at $x = a$

B. discontinuous only for $x > a$

C. a constant function when $x > a$

D. strictly increasing when $x < a$

Answer: C

Solution:

The function $f(x) = \frac{|x-a|}{x-a}$ can be analyzed by considering different cases for the value of x relative to a.

For $x > a$, the expression $|x-a|$ simplifies to $x-a$. Thus, the function becomes:

$f(x) = \frac{x-a}{x-a} = 1$

For $x < a$, the expression $|x-a|$ simplifies to $-(x-a)$, which means the function is:

$f(x) = \frac{-(x-a)}{x-a} = -1$

Thus, the function $f(x)$ can be rewritten as a piecewise function:

$$f(x) = \begin{cases} 1, & \text{if } x > a \\ -1, & \text{if } x < a \end{cases}$$

Based on this function:

$f(x)$ is a constant function (equal to 1) when $x > a$.

It is discontinuous at $x = a$ because the limits from the left and the right of a do not match. Specifically, the left-hand limit approaches -1 while the right-hand limit approaches 1, and thus the function does not have a limit at this point.

In summary, $f(x)$ is not continuous at $x = a$ and is constant when $x > a$.

Question10

If $f(x) = 3x^{15} - 5x^{10} + 7x^5 + 50\cos(x-1)$, then $\lim_{h \to 0} \frac{f(1-h)-f(1)}{h^3+3h}$

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Options:

A. -25

B. 25

C. -10

D. 10

Answer: C

Solution:

Given the function:

$f(x) = 3x^{15} - 5x^{10} + 7x^5 + 50\cos(x-1)$

We need to evaluate the limit:

$\lim\limits_{h\to 0} \frac{f(1-h)-f(1)}{h^3+3h}$

This expression is initially an indeterminate form $\frac{0}{0}$, so we can apply L'Hôpital's rule.

Using L'Hôpital's rule, the limit becomes:

$\lim\limits_{h\to 0} \frac{-f'(1-h)}{3h^2+3}$

We then evaluate this limit by substituting $h = 0$:

$\frac{-f'(1)}{3}$

To find $f'(x)$, we differentiate $f(x)$:

$f'(x) = 45x^{14} - 50x^9 + 35x^4 - 50\sin(x-1)$

Substitute $x = 1$ into $f'(x)$:

$f'(1) = 45(1)^{14} - 50(1)^9 + 35(1)^4 - 50\sin(0)$

$f'(1) = 45 - 50 + 35 + 0$

$f'(1) = 30$

Substitute back into the limit expression:

$\frac{-30}{3} = -10$

Thus, the value of the limit is -10.

Question11

If the function $f(x) = \begin{cases} \frac{(e^{kx}-1)\sin kx}{4\tan x} & x \neq 0 \\ P & x = 0 \end{cases}$ **is differentiable at** $x = 0$**, then**

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Options:

A. $P=0, f'(0)=\frac{k^2}{4}$

B. $P=0, f'(0)=-\frac{1}{2}$

C. $P=k, f'(0)=-\frac{k^2}{4}$

D. $P=k, f'(0)=-\frac{1}{4}$

Answer: A

Solution:

To ensure the function $f(x)$ is differentiable at $x=0$, it must also be continuous at this point. Therefore, we need:

$$\lim_{x\to 0}\frac{(e^{kx}-1)\sin kx}{4\tan x}=P$$

Let's simplify the limit step-by-step:

Convert the expression using known limits:

$$\lim_{x\to 0}\frac{kx}{4}\left[\frac{e^{kx}-1}{kx}\right]\left[\frac{\sin kx}{kx}\right]\cdot\frac{kx}{x\left(\frac{\tan x}{x}\right)^2}$$

Recognize standard limits for small x:

$$\lim_{x\to 0}\frac{e^{kx}-1}{kx}=1$$

$$\lim_{x\to 0}\frac{\sin kx}{kx}=1$$

$$\lim_{x\to 0}\frac{\tan x}{x}=1$$

Plug these into the equation:

$$\lim_{x\to 0}\frac{k^2x^2}{4x}=\lim_{x\to 0}\frac{k^2x}{4}=P$$

As $x\to 0$, the expression $\frac{k^2x}{4}\to 0$, so $P=0$.

Since $P=0$, let's find the derivative at $x=0$:

$$f'(x)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=\lim_{h\to 0}\frac{\frac{(e^{kh}-1)\sin kh}{4\tan h}}{h}$$

Applying the same logic and limits as before, the derivative becomes:

$$f'(0)=\frac{k^2}{4}$$

Therefore, $P=0$ and $f'(0)=\frac{k^2}{4}$.

Question12

If Rolle's Theorem is applicable for the function $f(x)=\begin{cases} x^p \log x, & x \neq 0 \\ 0, & x=0 \end{cases}$ on the interval $[0,1]$, then a possible value of p is

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Options:

A. -2

B. -1

C. 0

D. 1

Answer: D

Solution:

To apply Rolle's Theorem, we need the function to satisfy $f(a)=f(b)$ while $a \neq b$. For this function, consider :

$$\lim_{x \rightarrow 0} x^p \log x=0$$

Expanding the logarithm using Taylor series, we get :

$$\lim_{x \rightarrow 0} x^p\left[x-1-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}+\ldots\right]=0$$

If $p=0$, the limit does not approach zero.

Therefore, the minimum value of p that satisfies the condition for the limit to be zero is $p=1$.

Question13

If $\lim_{x \rightarrow 4} \frac{2x^2+(3+2a)x+3a}{x^3-2x^2-23x+60}=\frac{11}{9}$, then $\lim_{x \rightarrow a} \frac{x^2+9x+20}{x^2-x-20}=$

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Options:

A. -9

B. -4

C. $-\frac{1}{4}$

D. $-\frac{1}{9}$

Answer: D

Solution:

Given, $\lim\limits_{x \to 4} \frac{2x^2+(3+2a)x+3a}{x^3-2x^2-23x+60} = \frac{11}{9}$

Put $x = 4$

$= \frac{4}{32+12+8a+3a} \frac{64-32-92+60}{64+11a}$

$\because$ Numerator must be zero

$\Rightarrow \quad a = \frac{-44}{11} = -4$

Then, $\lim\limits_{x \to -4} \frac{x^2+9x+20}{x^2-x-20}$

$\lim\limits_{x \to -4} \frac{(x+5)(x+4)}{(x-5)(x+4)} \Rightarrow \frac{-4+5}{-4-5} = \frac{-1}{9}$

Question14

If the function

$$f(x) = \begin{cases} \dfrac{\tan(ax-1)}{x-1}, & \text{if } x \neq 1 \\ b, & \text{if } x = 1 \end{cases}$$

is continuous in its domain, then find the value of

$$6a + 9b^4.$$

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Options:

A.

284

B.

261

C.

214

D.

317

Answer: A

Solution:

$\because f(x)$ is continuous at $x=4$.

$$f(4)=\frac{64-125}{16-25}=\frac{61}{9}$$

$$f\left(4^{+}\right)=\frac{b^4-1}{4}=\frac{61}{9}$$

$$b^4=\frac{244+9}{9}=\frac{253}{9} \Rightarrow 9 b^4=253$$

$\because f(x)$ is continuous at $x=1$

So, $f\left(1^{+}\right)=f(1)=\frac{1-125}{1-25}=\frac{31}{6} \Rightarrow f\left(1^{-}\right)=a$

$\Rightarrow 6 a+9 b^4 \Rightarrow 31+253 \Rightarrow 284$

Question15

$\lim _{\theta \rightarrow \frac{\pi}{2}^{-}} \frac{8 \tan ^4 \theta+4 \tan ^2 \theta+5}{(3-2 \tan \theta)^4}=$

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Options:

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -4

D. 1

Answer: B

Solution:

$$\lim_{\theta \to \frac{\pi^-}{2}} \frac{8\tan^4\theta + 4\tan^2\theta + 5}{(3 - 2\tan\theta)^4}$$

$$= \lim_{\theta \to \frac{\pi^-}{2}} \frac{8\sin^4\theta + 4\sin^2\theta\cos^2\theta + 5\cos^4\theta}{(3\cos\theta - 2\sin\theta)^4}$$

$$= \frac{8(1)^4 + 4(1)^2(0)^2 + 5(0)^4}{(3(0) - 2(1))^4} = \frac{8}{16} = \frac{1}{2}$$

Question16

Define $f : R \to R$ by $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x > 0 \end{cases}$

Then, the value of a so that f is continuous at $x = 0$ is

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Options:

A. 8

B. 4

C. 2

D. 1

Answer: A

Solution:

Given,

$$f(x)=\begin{cases}\frac{1-\cos 4x}{x^2} & , \quad x<0 \\ a & , x=0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , x>0\end{cases}$$

is continuous at $x=0$

$\therefore \quad \lim_{x\to 0^-} f(x)=f(0)$

$\Rightarrow \quad \lim_{x\to 0^-} \frac{1-\cos 4x}{x^2}=a$

$\Rightarrow \quad \lim_{x\to 0^-} \frac{1-(1-2\sin^2 2x)}{x^2}=a$

$\Rightarrow \quad \lim_{x\to 0^-} \frac{2\sin^2 2x}{x^2}=a$

$\Rightarrow \quad \lim_{x\to 0^-} 8\left(\frac{\sin 2x}{2x}\right)^2=a$

$\Rightarrow \quad 8(1)^2=a \Rightarrow a=8$

Question17

$$\lim_{x\to 0} \frac{3^{\sin x}-2^{\tan x}}{\sin x}=$$

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Options:

A. 0

B. 1

C. $\log_e 6$

D. $\log_e \frac{3}{2}$

Answer: D

Solution:

We have,

$$\lim_{x\to 0}\frac{3^{\sin x}-2^{\tan x}}{\sin x}\qquad\left(\frac{0}{0}form\right)$$

$$=\lim_{x\to 0}\frac{\ln 3\left(3^{\sin x}\cdot\cos x\right)-\ln 2\left(2^{\operatorname{an} x}\cdot\sec^2 x\right)}{\cos x}$$

$$=\frac{\ln 3\cdot 1-\ln 2\cdot 1}{1}=\ln 3-\ln 2=\ln\left(\frac{3}{2}\right)$$

Question18

If the function

$$f(x)=\begin{cases}\frac{\cos ax-\cos 9x}{x^2} & \text{, if } x\neq 0\\ 16 & \text{, if } x=0\end{cases}$$

is continuous at $x=0$, then $a=$

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Options:

A. ± 8

B. ± 7

C. ± 6

D. ± 5

Answer: B

Solution:

We have,

$$f(x)=\begin{cases}\frac{\cos ax-\cos 9x}{x^2} & \text{, if } x\neq 0\\ 16 & \text{, if } x=0\end{cases}$$

Now, $\lim_{x\to 0} f(x)=16 \quad \ldots$ (ii)

[$\because f(x)$ is continuous at $x=0$]

$\lim_{x\to 0}\frac{\cos ax-\cos 9x}{x^2}=16$

Take LHS

HS

$\lim_{x\to 0}\frac{\cos ax-\cos 9x}{x^2}$

$=\lim_{x\to 0}\frac{-a\sin ax+9\sin 9x}{2x}$

[using L.' Hospital's rule]

$=\lim_{x\to 0}\frac{-a^2\cos ax+81\cos 9x}{2}$

$=\frac{81-a^2}{2}=16 \quad$ [from, Eq. (i)]

$81-a^2=32\Rightarrow a^2=49\Rightarrow a=\pm 7$

Question19

If $f(x)=\begin{cases}\frac{8}{x^3}-6x & \text{, if } 0<x\leq 1\\ \frac{x-1}{\sqrt{x}-1} & \text{,if } x>1\end{cases}$ is a real valued function, then at $x=1$, f is

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Options:

A. continuous and differentiable

B. continuous but not differentiable

C. neither continuous nor differentiable

D. differentiable but not continuous

Answer: B

Solution:

We have,

$$f(x)=\begin{cases}\frac{8}{x^3}-6x, & \text{if } 0<x\le 1\\ \frac{x-1}{\sqrt{x}-1}, & \text{if } x>1\end{cases}$$

$f(1)=2$ and

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\left(\frac{8}{x^3}-6x\right)=2=\text{ LHL}$

$\text{RHL}=\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}\frac{x-1}{\sqrt{x}-1}$

$=\lim_{x\to 1^+}\frac{1-0}{\frac{1}{2\sqrt{x}}-0}=\frac{21}{1}=2$

$\because \quad f(l)=\text{LHL}=\text{ RHL}$

$\Rightarrow \quad f(x)$ is continuous at $x=1$

And for differentiability at $x=1$,

$\text{LHD } =\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$

$=\lim_{h\to 0}\frac{\frac{8}{(1-h)^3}-6(1-h)-f(2)}{-h}$

$=\lim_{h\to 0}\frac{8-6(1-h)^4-2(1-h)^3}{-h(1-h)^3}$

$=\lim_{h\to 0}\frac{0+24(1-h)^3+6(1-h)^2}{-[(1-h)^3-3h(1-h)^2]}$

$=-\lim_{h\to 0}\frac{24(1-h)+6}{1-h-3h}=-30$

$\text{RHD } =\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

$=\lim_{h\to 0}\frac{\frac{1+h-1}{\sqrt{1+h}-1}-2}{h}$

$=\lim_{h\to 0}\frac{h-2\sqrt{1+h}+2}{(\sqrt{1+h}-1)h}$

$= \lim_{h \to 0} \frac{1 - \frac{1}{\sqrt{1+h}}}{\frac{1}{2\sqrt{1+h}} \cdot h + \sqrt{1+h} - 1}$

$= 2 \lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h + 2(1+h) - 2\sqrt{1+h}}$

$= 2 \lim_{h \to 0} \frac{\frac{1}{2\sqrt{1+h}}}{1 + 2 - \frac{1}{\sqrt{1+h}}} = 2 \times \frac{\frac{1}{2}}{3-1} = \frac{1}{2}$

$\because$ LHD $\neq$ RHD

Hence, at $x = 1$, $f(x)$ is continuous but not differentiable.

Question20

$$\lim_{n \to \infty} \left[\left(1 + \frac{1}{n^2}\right) \left(1 + \frac{4}{n^2}\right) \left(1 + \frac{9}{n^2}\right) \ldots . (2) \right]^{1/n} =$$

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Options:

A. $16e^{-1}$

B. $2e^{\left(\frac{\pi-4}{2}\right)}$

C. $2 \log 2 - 1$

D. $2 + e^{\left(\frac{\pi-4}{2}\right)}$

Answer: B

Solution:

$$\mathrm{L} = \lim_{n \to \infty} \begin{bmatrix} \left(1 + \frac{1}{n^2}\right) \left(1 + \frac{4}{n^2}\right) \\ \left(1 + \frac{9}{n^2}\right) \ldots (2) \end{bmatrix}^{1/n}$$

$$= \lim_{n \to \infty} \left(\prod_{K=1}^{n} \left(1 + \frac{K^2}{n^2}\right) \right)^{1/n}$$

$$L = \lim_{n \to \infty} e^{\left(\frac{1}{n} \sum_{K=1}^{n} \ln\left(1 + \frac{K^2}{n^2}\right)\right)} \quad \ldots \text{(ii)}$$

Sum can be approximated by an integral in the limit as $n \to \infty$

$\Rightarrow \frac{1}{n} \sum_{K=1}^{n} \ln \left(1+\frac{K^2}{n^2}\right) \approx \int_0^1 \ln \left(1+x^2\right) dx$

Using integration by parts

$$\int_0^1 \ln \left(1+x^2\right) dx=\left[\ln \left(1+x^2\right) \cdot x\right]_0^1$$

$$-\int \frac{2 x^2+2-2}{1+x^2} d x$$

$=\ln 2-\int_0^1 2 d x+2 \int_0^1 \frac{1}{1+x^2} d x=\ln 2-2+\frac{\pi}{2}$

So, from Eq. (i), we get

$L=e^{\ln 2-2+\frac{\pi}{2}}=2^{\frac{\pi}{2}-2}=2^{\left(\frac{\pi-4}{2}\right)}$

Question21

$$\lim _{x \rightarrow 1}(1-x) \tan \left(\frac{\pi}{2} x\right)=$$

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Options:

A. $\pi / 2$

B. $2 / \pi$

C. 1

D. 0

Answer: B

Solution:

$\lim _{x \rightarrow 1}(1-x) \tan \left(\frac{\pi}{2} x\right)$

Let $1-x=t \Rightarrow x=1-t$

So, as $\Rightarrow t \rightarrow 0$

$$Now,\ \lim_{t \to 0} t \tan\left(\frac{\pi}{2}(1-t)\right)$$
$$= \lim_{t \to 0} t \tan\left(\frac{\pi}{2} - \frac{\pi t}{2}\right) = \lim_{t \to 0} t \cot\left(\frac{\pi t}{2}\right)$$
$$= \lim_{t \to 0} \frac{t}{\tan\left(\frac{\pi}{2} t\right)} = \frac{2}{\pi} \lim_{t \to 0} \left(\frac{\frac{\pi}{2} t}{\tan \frac{\pi t}{2}}\right)$$
$$= \frac{2}{\pi} \cdot 1 = \frac{2}{\pi}$$

Question22

If $f(9)=9$ and $f'(9)=4$, then $\lim_{x \to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3} =$

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Options:

A. 3

B. 4

C. 6

D. 9

Answer: B

Solution:

Given, $f(9)=9$ and $f'(9)=4$ and we have to find

$\lim_{x \to 9} \frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$

Apply L' Hospital Rule, we get

$$\lim_{x \to 9} \frac{\frac{1}{2\sqrt{f(x)}} \cdot f'(x)}{\frac{1}{2\sqrt{x}}} = \frac{\frac{1}{\sqrt{f(9)}} f'(9)}{\frac{1}{\sqrt{9}}}$$
$$= \frac{4/3}{1/3} = 4$$

Question23

$$\lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} =$$

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Options:

A. $\frac{1}{10}$

B. $-\frac{1}{10}$

C. $\frac{2}{5}$

D. $-\frac{2}{5}$

Answer: B

Solution:

$$\begin{aligned}
&\lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3} \\
&= \lim_{x \to 1} \frac{(2x-3)(\sqrt{x}-1)(\sqrt{x}+1)}{(2x^2+x-3)(\sqrt{x}+1)} \\
&= \lim_{x \to 1} \frac{(2x-3)(x-1)}{(2x+3)(x-1)(\sqrt{x}+1)} \\
&= \frac{(2-3)}{(2+3)} \times \frac{1}{(\sqrt{1}+1)} = -\frac{1}{5} \times \frac{1}{2} = \frac{-1}{10}
\end{aligned}$$

Question24

If a, b, c and k are non-zero real numbers and $\lim_{x \to \infty} x\left(a^{1/x} + b^{1/x} + c^{1/x} - 3k^{1/x}\right) = 0$, then $k =$

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Options:

A. 0

B. $(abc)^{1/3}$

C. $(abc)^{-1/3}$

D. 1

Answer: B

Solution:

To solve the given problem, we need to evaluate the limit:

$\lim_{x\to\infty} x\left(a^{1/x}+b^{1/x}+c^{1/x}-3k^{1/x}\right)=0$

We start by substituting $x=\frac{1}{y}$, which implies $y\to 0$ as $x\to\infty$. The expression becomes:

$\lim_{y\to 0}\frac{a^y+b^y+c^y-3k^y}{y}=0$

Using the known limit:

$\lim_{y\to 0}\frac{a^y-1}{y}=\log_e a$

This allows us to rewrite the expression as:

$\lim_{y\to 0}\left[\frac{a^y-1}{y}+\frac{b^y-1}{y}+\frac{c^y-1}{y}-3\frac{k^y-1}{y}\right]=0$

This simplifies to:

$\log_e a+\log_e b+\log_e c-3\log_e k=0$

Rewriting it gives:

$\log_e\left(\frac{abc}{k^3}\right)=0$

Thus, we have:

$\frac{abc}{k^3}=1$

Solving for k, we find:

$k=(abc)^{1/3}$
